Metaphor Studies Investigation in Mathematics Education: A Systematic Review

Ilham Falani\textsuperscript{1,a), Yetti Supriyati\textsuperscript{2,b), Jefri Marzal\textsuperscript{1,c), Dewi Iriani\textsuperscript{1,d), Gugun Manosor Simatupang\textsuperscript{1,e)\textsuperscript{b}}

\textsuperscript{1}Mathematics Education Study Program, Universitas Jambi, Jambi, Indonesia
\textsuperscript{2}Postgraduate Program, Universitas Negeri Jakarta, Jakarta, Indonesia

e-mail: \texttt{a)ilhamfalani@unja.ac.id}, \texttt{b)yetti.supriyati@unj.ac.id}, \texttt{c)jefri.marzal@unja.ac.id}, \texttt{d)dewi.iriani@unja.ac.id}, \texttt{e)gugun.manosor@unja.ac.id}

Abstract

Metaphor is a powerful cognitive tool to understand or develop mathematical concepts. Although metaphor has been progressively recognized in mathematics education, there is an inadequate systematic review of its application and achievement. Therefore, this study aims to investigate and map metaphors studies trends in mathematics education. Data were collected from a systematic review of 70 metaphor studies in mathematics education published from 2012–2021 using the PRISMA guidelines. The results showed the mapping of the metaphor studies’ attributes in mathematics education. Future research can use this information as the basis for determining the novelty of research related to metaphors. Furthermore, they can use this information to determine its potential and opportunities for future research. The primary limitation of this review is related to the sensitivity of the search strategy, which makes it impossible to cover relevant studies comprehensively.

Keywords: Atlas.ti, bibliometrics, metaphorical, research mapping, Vosviewer

INTRODUCTION

Presently, metaphor is progressively recognized in mathematics education (Lakoff, George, & Nunez, 2000; Macagno & Zavatta, 2014; Mun, 2013; Schlimm, 2013; Soto-Andrade, 2020). According to Ersozlu (2013), it is an expression of both abstract and concrete concepts. Metaphors are vigorous cognitive and highly functional tools for solving and developing mathematical problems and ideas (Soto-Andrade, 2020; Yildiz & Karadeniz, 2021).

According to Jensen (2008), metaphor improves brain-based learning, which is centered on factors such as neuroscience and cognitive development. This means how the learning occurs among students as they grow, mature socially, cognitively, and emotionally. Therefore, brain-based strategies need to be adopted in teaching to appeal to more learners (Bruer, 1999; Hileman, 2006).

Metaphors are used in developing mathematical concepts (Erdogan, Yazlik, & Erdik, 2014; Lakoff, 1992; Latterell & Wilson, 2016; Manya Raman & Öhman, 2013; Wright, 2017). It grounds the mathematical concepts in familiar domains and connects the various branches (Schlimm, 2016, 2013). Students tend to grasp challenging mathematical concepts metaphorically through knowledge construction based on their experiences (Ersozlu, 2013; Pradhan, 2018). For example, through the "polygons are paths" concept, students are able to make sense of static polygons. In addition, a picture of the staircase aids them in visualizing a function graph (Lakoff, George; Nunez, 2000; Pradhan, 2018).

\textit{Indonesian Journal of Mathematics Education, Vol. 6, No. 1, April 2023}
Metaphors enable the mapping of the inferential framework from its source to the target domain. It serves as a tool that simplifies opaque and abstract targets (Lakoff, George, Nunez, 2000; Turkkan & Uyar, 2016). Furthermore, metaphors elevate the students’ problem-solving, questioning, and critical thinking abilities (Hendriana, 2014, 2017; Setiani, Waluya, & Wardono, 2018).

Notwithstanding being a potential topic for analysis, there is no rife or overview of the current state of metaphor studies in mathematics education. There is an inadequate review on achievement and its application.

This study aims to investigate and map metaphor studies by analyzing the current state of the art through a systematic review. Moreover, providing an overview of its attributes is recommended for future analysis. Therefore, this led to the following research questions.

**RQ1.** What does the attributes summary reported on included studies based on bibliography analysis?

**RQ2.** What does the study focus summary on included studies?

**RQ3.** What does the methodologies summary report on the included studies?

**RQ4.** What does the summary of the outcome reported on included studies?

**METHOD**

This study is focused on metaphor in mathematics education. Systematic reviews were carried out based on the PRISMA Guidelines, including a 27-item evidence-based checklist (Moher et al., 2009). It is critical to provide detailed processes to ensure the review is repeatable and transparent (Higgins et al., 2019). Therefore, the search strategy, study selection, quality assessment, data extraction, analysis, and synthesis process are described in the following section.

**Search Strategy**

The most important step in carrying out a systematic review is accurately searching for all potentially relevant studies. Data were collected from a wide range of databases namely Scopus, Science Direct, Semantic Scholar, JSTOR, Wiley Online Library, SpringerLink, EBSCO, Microsoft Academic, Taylor & Francis, and Educational Resources Information Center (ERIC) due to their relevance in the subject area. This database was chosen because it has a large number of publications related to this research topic.

A keyword search strategy was adopted based on the important concepts, namely metaphor and mathematics education. Related and similar terms were identified using Merriam-Webster's Online Thesaurus (see Table 1).

<table>
<thead>
<tr>
<th>Metaphor</th>
<th>Mathematics</th>
<th>Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metaphor* (Metaphors, metaphoric, metaphorical)</td>
<td>Math* (maths, mathematics, mathematical)</td>
<td>Education* (Educational, educational School)</td>
</tr>
<tr>
<td>Metaphorical* (Metaphorical thinking, metaphorical ability)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The terms listed in Table 1 were further combined using the Boolean operators "AND" and "OR." Concatenation yielded a search string, used to source for abstracts related to relevant studies, such as ("Metaphor*" OR "Metaphorical") AND "Math*" AND "Education*". Meanwhile, studies containing the keywords in the title or abstract were obtained. To ensure that no relevant studies were missed, the search strategy was applied to the selected databases. The string was modified for various online databases while maintaining the logical order.
**Study Selection**

Various inclusion criteria were used to select the relevant studies, as shown in Table 2.

<table>
<thead>
<tr>
<th>Study Selection Criteria</th>
<th>Exclusion Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Published from 2012 to 2021</td>
<td>The paper was not published from 2012 to 2021</td>
</tr>
<tr>
<td>Published in English</td>
<td>The paper was not published in English</td>
</tr>
<tr>
<td>The focus of the study is a metaphor in mathematics education</td>
<td>The result is in the form of an editorial, generic, or book review</td>
</tr>
<tr>
<td>Follows empirical research methods</td>
<td>Incomplete journal article/conference articles/thesis</td>
</tr>
<tr>
<td>Full-text journal article/conference articles/thesis</td>
<td>The study is not focused on metaphor in mathematics education</td>
</tr>
<tr>
<td>The study does not provide a detailed empirical research design and data analysis</td>
<td>The study does not provide a detailed empirical research design and data analysis</td>
</tr>
</tbody>
</table>

*Include if the study meets ALL the inclusion criteria

To ensure that the research chosen for the systematic review is research that exclusively focuses on the study theme, the inclusion and exclusion criteria’s above must be explicitly stated.

After obtaining all the databases’ results, duplicate publications were discarded before applying the selected filter. The irrelevant studies retrieved due to poor search engine performance were eliminated with the inclusion criteria and proofreading their titles and abstracts. Figure 1 shows the steps used in selecting relevant studies.

The search terms in the databases were generated from 514 studies, which were screened by two authors to eliminate duplicate entries and determine the titles and abstracts. The screening excluded 174 articles, while the remaining 340 were analyzed in the next stage. The inclusion and exclusion criteria applied to the abstracts led to the elimination of 260 articles. Furthermore, full-text screening was used to eliminate 20 more articles, while the remaining 70 were included in the review. Therefore, 70 articles needed to be read and synthesized in full-text and qualitative analysis, respectively.

![Figure 1. Search and Selection Summary](image)

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**Quality Assessment**

The included studies’ quality is an essential aspect of a systematic review. This was evaluated using the following criteria.

**Research Design Description**

Any research found lacking detailed empirical design and analysis during data extraction was removed from the included studies. These were examined to ensure that a comprehensive description of the research aims, participants’ details, methodology or design, findings, and conclusions were included.

**Publication Outlet**

The SCImago journal ranking (SJR) and its Impact factor were used to assess the quality of the publication outlets. The majority of the included studies were published in journals. Compared to other publication outlets, the Journal of Physics Conference Series had the most relevant articles on the topic.

**Impact of the Included Studies**

Furthermore, the impacts of the included studies were analyzed. Citations of Google scholar indicate the importance of a study in the scientific community, which were analyzed to determine its influence on the existing research. The design and publication outlet of these studies ranges from acceptable to quality publications.

**Data Extraction**

The listed attributes, namely publication year, authors, title, outlets’ detail, and type (conference or journal), keywords, aims, research methodology, and research location, participants’ details, sample size, study focus, outcomes, findings, limitations, and conclusion were gleaned from the included studies. These were coded and analyzed to produce a reported summary of the research questions. In addition, Atlas.ti was used for coding. The initial author extracted and coded the first level of attributes, which was continuously verified by another. At the end of the data coding stage, the results and disagreements were discussed.

**Data Analysis and Synthesis**

The purpose of this study is to create a map of attributes related to metaphors in mathematics education. Bibliometric analysis was used to assess future scientific research opportunities. It utilized metadata from 70 included studies or documents. These metadata are extracted from various databases. All information is exported to CSV format for data analysis purposes, while the metadata is evaluated using Vosviewer to construct its visualization.

Furthermore, Atlas.ti was used for analysis and synthesis and to code all attributes in PDF files to answer the research questions. The outcomes and study foci were analyzed and synthesized iteratively by the first author, then verified by another. Figure 2 shows the steps used for analyzing and synthesizing the data.

![Figure 2. Data Analysis Process Using Atlas.ti](image-url)
report for each included study. Third and fourth identifies and classifies the multiple focuses and outcomes of each study, respectively. However, in this research, the outcomes are classified into 4 categories, namely Not Achieved (when the anticipated outcomes were not achieved), Inconclusive (when there was no information on anticipated outcomes or it was not clear that they had been achieved), Achieved (if the stated outcomes were achieved), and Mixed (some outcomes were achieved and others were not).

RESULTS AND DISCUSSION
This section is a summary of the analyzed 70 included studies regarding the research questions.

RQ1. What does the attributes summary reported on included studies based on bibliography analysis?

Frequency analysis of study attributes
The 70 included studies were published between 2012 and 2021.

In 2018, there was a significant increase in studies carried out on metaphor in mathematics education. Meanwhile, in 2021, the least amount was discovered. Figure 3 shows the classification of included studies with respect to their publication year.

The included studies type consists of 3 categories. However, journals or articles belong to the category with the highest number (see Figure 4).

Figure 3. Number of Publications Per Year

Figure 4. Distribution of Studies Type
Geographical distribution

The included studies were carried out in a variety of cultural settings around the world. However, this review only involved those that were published in English. This aims to facilitate researchers in conducting a review of the included studies, because these studies use international languages. Determination of inclusion and exclusion criteria in a systematic review is only studies that use English, and does not consider research published in other languages, as well as article texts that are not easy to obtain so that it has the potential to cause bias (Magarey, 2001). Figure 5 shows the countries where these publications were performed. The maximum number of studies on metaphor in to as "science mapping," based on the metadata extracted from the 70 included studies.

![Geographical Distribution of Included Studies](image)

Figure 5. Geographical Distribution of Included Studies

Metadata Analysis

The Vosviewer was used to generate the bibliometric visualization (see Figure 6), often referred mathematics education was dominated by those carried out in Turkey, followed by the United States.

![Bibliometric Network Visualization](image)

Figure 6. Bibliometric Network Visualization
Each circle in Figure 6 indicates a term or keyword that frequently appears in the abstract and title of the study. Its size represents the number of publications that used the terms in the research title and abstract. Furthermore, the more relevant a term or keyword is in an article, the larger the circle size. Adjacent terms that frequently occur tend to be closer to each other during visualization. Moreover, it was discovered that the metadata of 70 included studies are grouped into 6 clusters categorized into 3 dominant and 3 minority clusters, each of which is identified based on color.

In Figure 7, the color of the node represents the keyword and year of publication. The brighter the node indicates the more recent year the keyword is researched.

**Figure 7. Bibliometric Overlay Visualization**

Figure 8 indicates the occurrence rate or keyword density in the metadata. The brighter the keyword, the more frequently it is found in the metadata. This indicates that the longer the topic is discussed, the darker the color on the node.

**Figure 8. Bibliometric Density Visualization**
After identifying the mapping and clustering of metaphor studies in mathematics education based on the keywords, Vosviewer was used to generate a research trend mapping based on the studies’ year of publication. However, from the analyzed metadata imported into VOSviewer, overlay visualization was also generated. The information obtained is used to detect and identify the most recent condition of metaphors in the past 5 years, as shown in Figure 7. That the study of metaphors in mathematics education related to geometry, problem-solving, metaphor analysis, and teacher training has been discussed for a longer period than topics related to metaphorical thoughts, perception, and ethnography. The topics that appear in the visualization usually become research trends in this field.

<table>
<thead>
<tr>
<th>Study Focus</th>
<th>Description</th>
<th>Frequency</th>
<th>Paper ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual of Metaphor</td>
<td>To investigate its concept and philosophy</td>
<td>10</td>
<td>P2, P11, P34, P36, P45, P52, P54, P58, P62, P70</td>
</tr>
<tr>
<td>The Use of Metaphor as a cognitive tool</td>
<td>To investigate its use as a cognitive tool for understanding mathematical thoughts and beliefs.</td>
<td>20</td>
<td>P1, P8, P9, P12, P25, P26, P28, P34, P35, P36, P39, P45, P51, P52, P55, P61, P63, P68, P69</td>
</tr>
<tr>
<td>Type of Metaphor</td>
<td>To investigate the types used in mathematics</td>
<td>3</td>
<td>P10, P33, P37</td>
</tr>
<tr>
<td>Effectiveness of the applied Metaphor</td>
<td>To investigate the effectiveness of applying it in learning</td>
<td>10</td>
<td>P16, P17, P30, P42, P43, P44, P56, P57, P59, P66</td>
</tr>
<tr>
<td>Teachers' Perceptions</td>
<td>To investigate the Teachers’ Perception of mathematics through metaphor</td>
<td>19</td>
<td>P4, P8, P10, P14, P15, P18, P19, P24, P31, P32, P38, P41, P46, P47, P48, P49, P50, P53, P64</td>
</tr>
<tr>
<td>Students' Perceptions</td>
<td>To investigate the Students' Perceptions of mathematics through metaphor</td>
<td>11</td>
<td>P5, P7, P17, P20, P23, P29, P40, P55, P60, P65, P67</td>
</tr>
<tr>
<td>Parents' Perceptions</td>
<td>To investigate the parents' Perceptions of mathematics through metaphor</td>
<td>1</td>
<td>P22</td>
</tr>
<tr>
<td>Technology Implementation</td>
<td>To investigate the implementation of technology with a metaphorical approach in terms of learning mathematics</td>
<td>4</td>
<td>P3, P28, P42, P59</td>
</tr>
<tr>
<td>Learning Engagement</td>
<td>To explores the metaphor approach that facilitates the development of learning activities with the potential to increase both students and teachers’ engagement and confidence</td>
<td>1</td>
<td>P3</td>
</tr>
<tr>
<td>Measuring of metaphorical ability</td>
<td>To measure the metaphorical ability</td>
<td>1</td>
<td>P21</td>
</tr>
<tr>
<td>Curriculum Development</td>
<td>To investigate the development of curriculum integrated with Metaphorical Approach</td>
<td>2</td>
<td>P27, P13</td>
</tr>
<tr>
<td>Instructional Design</td>
<td>To investigate the metaphor instructional design in terms of assisting students</td>
<td>1</td>
<td>P6</td>
</tr>
</tbody>
</table>

The identified focus of the included studies is divided into 11 major categories based on the review results. Most studies usually focus on using metaphor as a cognitive tool, followed by investigating the teachers’ perceptions of mathematics through metaphor (teachers’ perceptions). In addition, understudied research topics are greatly emphasized. It is interesting to investigate these foci as trends in metaphor in future mathematics education research.
RQ3. What does the methodologies summary report on the included studies?

The included studies applied various approaches to the research design, including quantitative, qualitative, and mixed methodologies. According to the frequency analysis, the qualitative method is the most used approach. Fig. 9 depicts the methodologies and research paradigms used in the included studies, with 74% realized using qualitative methods, while 16% adopted a quantitative approach, and the remaining 10% applied a combination of both. Furthermore, Figure 9 depicts the classification of research methodologies within each paradigm as explicitly stated in included studies. The most used ones are phenomenology (qualitative) and experimental designs (quantitative). The poor prevalence of mixed methods (6%) was notable.

![Figure 9. Methodologies and Research Paradigms](image-url)

Figure 9 shows that most mathematical studies do not specify the sub-domain of investigation. Geometry and algebra were frequently studied compared to other non-specified sub-domains, followed by calculus, functions, statistics, and arithmetic.

![Figure 10. Sub-domain Involved in Mathematics](image-url)
The 70 studies that were reviewed involved a wide variety of participants or research objects are shown in Figure 11. These include mostly prospective teachers and secondary school students. The sample size is divided into 9 categories based on the number of participants, as shown in Figure 12. This indicates that most studies were carried out with less than 50 or 100 participants. However, only a few were carried out with more than 300 participants.

Figure 11. Number of Participants Involved in Studies

Figure 12. Sample Sizes in Studies

RQ4. What does the summary of the outcome reported on included studies?

It was reported whether the stated objectives were achieved. The term "study outcomes" was frequently used in this research. The results were classified into the following categories, namely achieved (if the research findings can answer all research questions), not achieved (if the study indicated that the stated expected outcomes were not met), inconclusive (if the study did not provide any information about anticipated outcomes or it was not clear that the anticipated outcomes had been achieved), and mixed (if some outcomes were achieved and others were not).
According to Figure 13, 62 (89%) studies were Achieved, 6 (9%) were Inconclusive, and only 2 (2%) were categorized as Not Achieved study outcomes.

Metaphor studies in mathematics education have developed progressively. The aim of this research is to determine its state of the art. Mapped research is needed to discover the potential and opportunities regarding topics that need to be investigated in the future. Through the bibliometric analysis that was carried out, this research tries to visualize the network. Bibliometric analysis is a technique used to provide a network structure that addresses questions, such as the main topics in a particular field of study, its relations with one another, and its development over time (Waltman, van Eck, & Noyons, 2010).

The use of metaphor as a cognitive tool to understand the mathematical concept was the most frequently identified focus. Several studies included in this review discovered that the role of metaphor in the educational process is to aid in mathematical meaning-making and knowledge construction. It also assists students in learning ways to communicate mathematical concepts (Núñez & Lakoff, 2013). Besides, many studies have implemented metaphors in several mathematical sub-domains, as shown in Figure 10. González (2013) carried out research on the implementation of metaphor with a prototypical image to assist students in remembering a set of theorems. Patel, McCombs, and Zollman (2014) investigated the students’ metaphorical reasoning regarding the limit concept in calculus. Dawkins (2012) analyzed the implementation of metaphor as the path to interpret the sequence convergence.

Based on the aforementioned studies, metaphors are used in the mathematical sub-domain context, which is abstract and difficult to explain by the teachers. On the other hand, students find it difficult to understand the abstract context. Therefore, the use of metaphors is the appropriate solution to solve these problems. The human memory also struggles to understand abstract data. A perceived abstract concept is embodied in such a way that it is precisely understood by the brain. The use of metaphor is an essential tool that aids in concretizing this concept (Doğan & Sönmez, 2019). It is also regarded as an effective mental tool for comprehending and explaining a highly abstract, complex, or conceptual phenomenon (Kiliç & Yelken, 2012). Metaphors are used to transform abstract concepts that are difficult to understand through analogy into concrete ones that have already been encountered (Gecit & Gencer, 2011). It aids in revealing certain effective situations with fewer words (Doğan & Sönmez, 2019). Metaphors used in teaching are useful for connecting one conceptual situation to another as well as play an important role in solving many problems (Craig, Diamond, & Shih, 2020).
The next most commonly identified focus is the teachers' or students' perceptions of mathematics through metaphor. It analyzes different views and perspectives. Moreover, metaphors also contribute to revealing the way these concepts are perceived (Cemalettin Yildiz & Hacisalihoglu Karadeniz, 2021; Cemalettin Yildiz & Karadeniz, 2018). Based on this reason, it is considered an important tool that reflects past, present, and future perceptions.

In addition, several foci of included studies have been identified, although in small numbers. However, an intriguing discovery from this review relates not to the most prominent focus rather to that, which is absent from the included studies because the unidentified foci indicate the potential for future research.

There is a need to deconstruct its manifestations in curriculum materials to highlight the influence on learning due to the importance of metaphor. This research focus is rare, especially the studies included in this review (Choppin et al., 2015; Dietiker, 2015). Furthermore, only English-language literature was considered.

This review attempted to ensure that the results strictly adhered to the guidelines, although it does not protect against research bias. Furthermore, during coding, research bias has the potential to occur. More critical analyses are required to address this threat. The coders independently selected studies and compared the results. The authors worked closely together to ensure high interrater reliability. The discordance studies were collectively rated. Discussions were held to achieve consensus. Every point of contention was documented and debated until an agreement was reached. Only codes from the agreement were entered into the final coding table.

CONCLUSION

In conclusion, this research analyzed the findings of 70 included studies on metaphor in mathematics education published between 2012 and 2021. The aim is to investigate and map Furthermore, the investigation relating to the extent the metaphoric approach facilitates the development of learning activities to increase student or teacher engagement and confidence needs to be further analyzed (learning engagement). As a tool to assess the metaphorical ability and investigate its instructional material design to assist student learning, as earlier mentioned, are effective and aids in better understanding and defining of scientific concepts. It is therefore important to develop future research.

Limitations of the study

Irrespective of the fact that the systematic review adhered to strict guidelines to ensure the completeness of the sample, some publications were not included in the search engine. This was caused by different algorithms and search procedural requirements in each database. Another factor that tends to influence study selection is that only 10 large databases were searched during the systematic review. This included all works in the field, metaphorical attributes to see the solutions achieved and their classifications (focus, methodologies, participants, and outcomes). This review provided insights into the focus, methodologies, participants, and outcomes of preliminary studies. The most frequently stated focus was the use of metaphor as a cognitive tool to understand mathematics and the teachers' perception. Meanwhile, the commonly identified methodology was phenomenology, and most of the participants were prospective teachers and secondary school students.

An intriguing finding from this review does not relate to qualities reported in preliminary studies rather to those completely excluded. Many of the studies included in this review do not appear to consider the evolution of metaphor instructional design in mathematics learning. Furthermore, future research needs to consider the development of standardized instruments for assessing metaphor abilities.

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Núñez, R., & Lakoff, G. (2013). The cognitive foundations of Mathematics: The role of


http://dx.doi.org/10.46827/ejes.v0i0.863


## APPENDIX A. LIST OF INCLUDED STUDIES

<table>
<thead>
<tr>
<th>Paper ID</th>
<th>Citation</th>
</tr>
</thead>
<tbody>
<tr>
<td>P8</td>
<td>Alsulami, N. M., &amp; Taylor, P. C. (2012). Awakening Saudi mathematics teachers to their traditional conception of teaching and learning: The prospect of metaphor as a tool for reconceptualizing professional practice Our aim in this paper is to explore beliefs about teaching and learning held by. In <em>The 7th International Conference on Science, Mathematics and Technology Education at Sultan Taboos University, Muscat, Oman</em>, 2012, 1–9.</td>
</tr>
</tbody>
</table>

*Indonesian Journal of Mathematics Education, Vol. 6, No. 1, April 2023*


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