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Students' Combinatorial Thinking Error in Solving Combinatorial Problem

Gusti Uripno[1,a], Tatag Yuli Eko Siswono[1,b], Endah Budi Rahaju[1,c], Arief Budi Wicaksono[2,d]
[1]Universitas Negeri Surabaya, Jl. Lidah Wetan, Lakarsantri - 60213, Surabaya, (031) 8280009
[2]Universitas Tidar, Jl. Kapten Suparman No.39, Potrobangsan, Kec. Magelang Utara, Kota Magelang, Jawa Tengah 56116, (0293) 364113
e-mail: [a]gusti.21008@mhs.unesa.ac.id, [b]tatagsiswono@unesa.ac.id, [c]endahrahaju@unesa.ac.id, [d]ariefbudiw@untidar.ac.id

Abstract

Combinatorial thinking errors describe students' difficulties and obstacles in solving combinatorial problems. This study aims to describe the errors experienced by students in solving combinatorial problems in terms of combinatorial thinking processes. This research involved two subjects who were 12th grade high school students at a school in Gresik, Indonesia. The students have already taken a combinatorics course. Data collection was conducted using the think-aloud observation method and task-based interviews. Both methods of data collection were conducted to validate the data using the triangulation method. The two subjects experienced similar errors. The research shows that the filling slots method is a simple and easy way for students to understand, but problems arise when students cannot understand the meaning of the questions and input the correct numbers for the problem. The combinatorial thinking error includes the general counting process and vertical upward formulas or expressions. The general counting process error is generating a number that represents the given aspects of the problem and the vertical upward formula/expression is identifying the concept that fits the problem. This research suggests enhancing students understanding of number representation when teaching the filling slot method. The teacher should illustrate some of the multiplication rule and addition rule examples to help students distinguish between these two fundamental rules. Further research is needed to provide solutions to the constraints experienced by students in solving combinatorial problems.

Keywords: Combinatorial thinking, error, high school students

INTRODUCTION

Combinatorial thinking is a very important reasoning process in building students' knowledge and learning experiences (Hidayati et al., 2020). Combinatorial thinking is a basic ability that can develop critical thinking skills and problem solving abilities (Hidayati et al., 2019). Combinatorial thinking has an important role in learning mathematics. Combinatorial thinking is one of the abilities that support solving mathematical problems and mathematical modeling (Medová et al., 2020). The importance of combinatorial thinking is driven by technological developments that make combinatorics a very important topic in the world of science and technology (Salavatinejad et al., 2021). So that the importance of combinatorial thinking is caused by the needs of human life, especially in the fields of technology and science, which are very dependent on the topic of combinatorics itself.

Combinatorial thinking can be seen as a person's mental activity to solve discrete problems, which include combinatorics problems. Combinatorial thinking is also seen as a process in finding solutions to discrete problems (Ammamiarihta et al., 2017). Combinatorial thinking is one's thinking ability in solving combinatorics problems, which include techniques and strategies (Rezaie & Gooya, 2011). Combinatorial thinking is a way for someone to solve combinatorics problems that includes enumeration rules (Lockwood, 2013). Combinatorial thinking can be interpreted as a process of solving problems related to the concept of combinatorics (Aini et al., 2018). Combinatorial thinking is a type of thinking that involves solving interactive problems that exist in daily life (Wu & Molnár, 2018).

Lockwood formulates a combinatorial thinking framework in three parts (Lockwood, 2013). The framework compiled by Lockwood is divided into three interconnected parts. The parts mentioned by Lockwood include the counting process, set of outcomes, and formula/expression. The counting process is defined as an enumeration process, which includes where a number is expressed and how to calculate it from the problems described. A set of outcomes is defined as the process of collecting or mentioning each object in question. This process can be demonstrated by making diagrams, charts, or tables. Formula/ expression is defined as the process of stating a mathematical formula/expression. Mathematical expressions can be in the form of notation or number operations.

Apart from Lockwood's combinatorial thinking model, there are other combinatorial thinking models developed by Salavatinejad, Alamolhodaei, & Radmehr (Salavatinejad et al., 2021). The model consists of three main parts, which include horizontal movement, upward vertical movement, and downward vertical movement. Horizontal movement allows students to work around the problem itself. This allows students to try to break down the problem into sub-problems, make an analogy with another problem, or check answers that count excessively or insufficiently. The vertical upward movement explains how students try to make the problem more concrete. Abstract problems will be represented by simple examples or visual representations. Vertical upward movement allows students to determine combinatorics concepts that match the given problem. This section describes how a problem is found in its general or mathematical form.

Preliminary research conducted by researchers showed several high school students' mistakes in solving combinatorics problems, which were classified in each part of the Lockwood thinking model (Lockwood, 2013). Another study found that high school students were inappropriate in using the combinatorics formula for problems with many code arrangements (Uripno, 2020). Dwinata revealed that out of 32 subjects, 31.25% of students could solve combinatorics questions, while 37.5% of students could not solve combinatorics questions with a low level of difficulty (Dwinata, 2019). There are several obstacles experienced by students in solving combinatorics problems, namely: difficulty receiving information, errors in classifying objects, procedural errors, and errors in applying concepts to existing problems (Astuti et al., 2017). Other research shows that students cannot see combinatorics problems in general (Uripno & Rosyidi, 2019). The iterations carried out by students have not been able to touch the general form of the given problem. Students' difficulties in solving word problems related to the principle of combinatorics. These difficulties include difficulties in understanding the content of the questions, difficulties in understanding concepts, difficulties in using appropriate procedures, and difficulties in the factorial principle (Sinaga et al., 2021).

This study aims to describe the errors experienced by students in solving combinatorial problems in terms of combinatorial thinking processes. The combinatorial thinking framework is the conclusion of two combinatorial thinking models by Lockwood (Lockwood, 2013) and Salavatinejad et al. (Salavatinejad et al., 2021). Based on the two models, similarities were found, which were then concluded in the research framework.

METHOD

This study aims to find students' combinatorial thinking errors. The researcher will examine students' combinatorial thinking processes and highlight combinatorial thinking errors and then the researcher will describe these errors. Descriptive research is research that aims to describe and interpret information about symptoms that arise when research is conducted (Siswono, 2019). So, based on these descriptions, this research is descriptive research with a case study type.

This research involved two subjects who were 12th grade high school students at a school in Gresik, Indonesia. The students have already taken a combinatorics course. Subjects were selected based on the results of the answers and the consideration of the school teacher. The two students are 12th grade high school students who experience errors in solving combinatorial problems that have been given by the researcher and have good communication. This was conducted because data collection was done using the think-aloud observation method and task-based interviews. Both methods of data collection were conducted to validate the data using the triangulation method.

The approach used in this research is qualitative. The data in this study will be analyzed and presented qualitatively. The stages that will be carried out in analyzing the data refer to the data analysis techniques namely data reduction, data presentation, and conclusions (Miles & Huberman, 1984). The questions given to the subject are

> *"Andi, Aril, Ilham and seven of their friends will be on vacation using two cars, namely a family car and a mini car. The family car can only accommodate six people (one driver and five passengers). The mini car can accommodate four people (one driver and three passengers). If only Andi, Aril and Ilham can be drivers, how many choices of seating plans can you make?"*

RESULTS AND DISCUSSION

The discussion of the research results will refer to several categories that are inferred from the Lockwood (Lockwood, 2013) and Salavatinejad et al. (Salavatinejad et al., 2021) models, namely as follows:

1. Generate a number that represents the given aspects of the problem (CU1);
2. Calculate created list (CU2);
3. Calculating the results of operations on numbers that have been compiled (CU3);
4. Check the suitability of the calculated list (CH1);
5. Check the suitability of the arithmetic operation (CH2);
6. Divide the problem into several cases (SH1);
7. Make a list of events that are asked (SH2);
8. Checking the compatibility of cases with the list made (SH3);
9. Create a chart, table, or chart (SVb1);
10. Raises issues that are relevant to the given problem (FH1);
11. Generate formulas based on problems that are relevant to the given problem (FH2);
12. Check the suitability of the formula through a simple example (FVb1);
13. Identify the concept that fits the problem (FVa1);
14. Arrange the general form of the given problem (FVa2).

The discussion will be conducted by dividing the description into two sub-chapters for each subject.

1. Subject 1

The results of the subject's work can be seen in Figure 1. Based on these results, interviews were conducted with the subject. The results of this interview show several results, including combinatorial thinking processes and their mistakes.

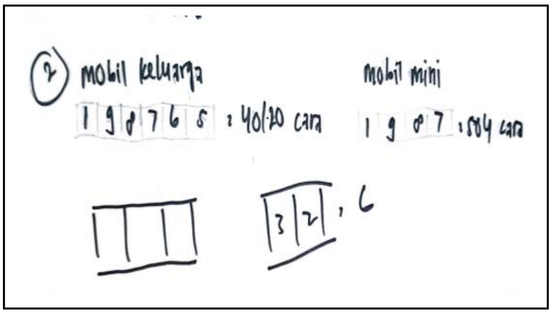

Figure 1. Subject 1 Task Result

The subject initially determines the number of people to depart and who will be the driver. This is shown by the quotation from the subject who said that the group consisted of Andi, Ariel, Ilham, and seven of their friends, showing that there were as many as 10 people who would depart. While the driver can only be filled by three children. This stage can be included in the Generate a number that represents the given aspects of the problem (CU1) category.

Then based on the results of the interviews obtained based on the following quotations.

Researcher: What do you think about next when you do the work?
Subject 1: several boxes
Researcher: What do you mean?
Subject 1: Possibility questions as usual using boxes.
Researcher: Why are there four boxes?
Subject 1: There are four numbers that will be used as passwords, so I will give them four boxes.

Based on the quote, it can be seen that the subject then arranges boxes according to the number of each car, namely 6 and 4. So this process can be categorized as Create a chart, table, or chart (SVb1).

Figure 1 shows that the subject's slot filling method has errors. This is also indicated by the interview transcript. The first error is that the first box representing the driver's box is filled with the number one. The subject filled one in the driver's box for a family car and a mini car. The subject assumes that one driver is selected, so the number entered in the first slot is one. This error can be categorized as a Generate a number that represents the given aspects of the problem (CU1) error.

The slot filling error reappeared when the subject filled the next slot. A subject fills the next slot with the number 9 when the slot should be filled with the number 8 because two children have been chosen to be drivers. The subject argument can be seen in the following quote.

"Then three children means one, sir, only one can be a driver, so I write one in the left corner and the rest of his friends means 1, 9, 8, 7, 6, 5 if multiplied, it means the result is 40.120 ways"

Based on the quote, the subject stated that the next slot was filled with nine because one child had been chosen as a driver. This category is an error category Generate a number that represents the given aspects of the problem (CU1). The quote is an explanation of the subject of filling the family car slot. Nonetheless, the same error occurred when the subject filled in the slots for mini car boxes.

The subject then multiplied the numbers in each slot. This process can be categorized as Identify the concept that fits the problem (FVa1). Then, by counting the formula, the subject finds the result. This process is known as Calculating the results of operations on numbers that have been compiled (CU3). Even though this step was correct, the subject was wrong in the previous step, which resulted in an error in the results obtained. The mistakes made by the subject were also found in the next steps, which can be seen in the following excerpts.

Researcher: What's the final result?
Subject 1: 40.624
Researcher: Where did you get it from?
Subject 1: Add up this one with this one

Based on the quote, the subject said that the result obtained was the sum of the multiplication results for the family car box and the mini car box. The multiplication rule is a more precise rule than the addition rule. This can be categorized as Identify the concept that fits the problem (FVa1) and the calculation process of the formula can be categorized as Calculating the results of operations on numbers that have been compiled (CU3).

The flow of the process on the subject can be sketched out based on the description of the completion of the subject as follows:

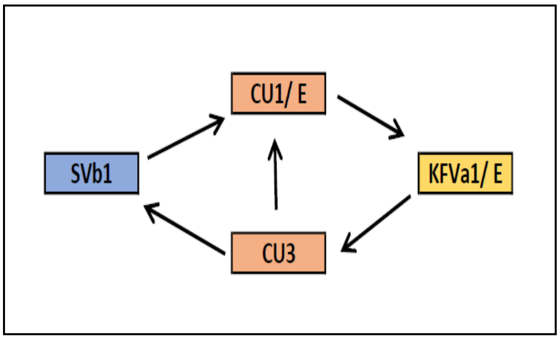

Figure 2. Flowchart of Subject 1 Process

2. Subject 2

The results of subject 2's work can be seen in Figure 3 below.

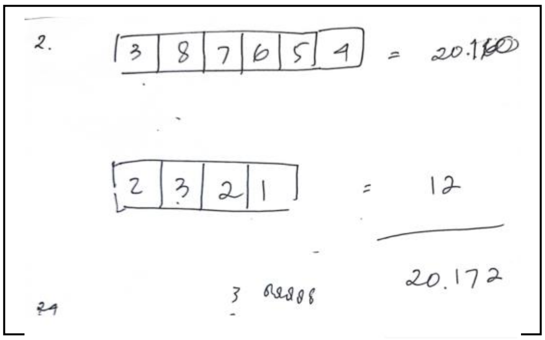

Figure 3. Subject 2 Task Result

The subject starts solving by bringing out the known numbers in the problem. One subject identified that there were three children who could become drivers and there were six people in family cars and four people in mini cars. This initial step can be categorized as Generate a number that represents the given aspects of the problem (CU1). Furthermore, the subject will make boxes, which can be seen in Figure 3 and the following excerpt.

Researcher: So what are you going to do next?

Subject 2: I think of it in a checkerboard way

Researcher: Is there any other way than checkers?

Subject 2: Yes, but I'm not sure because I don't really understand.

Based on the quote, it can be concluded that the subject made boxes to help count. In addition, the subject does not know any other way than by representing it in boxes. This process can be categorized as Create a chart, table, or chart (SVb1).

Then the subject fills each box with a certain number. The first box is filled with the number three because this box is considered the driver for the family car because there are three drivers. Then the subject filled in two for the mini car driver's box with the number two because the driver's choice had already been chosen as one in the family car. This process can be seen in the following quote.

"I filled the first box with three because there was a driver's choice and the first box for the mini car, I filled two. Then, because the mini car can carry six people and one driver has taken it, there are only 5, 4, 3, 2, 1 left. Meanwhile, the family car can carry four people and if they have become one driver, there are only 3, 2, 1 left."

Based on these quotes, it can be seen that the subject prioritizes family cars to choose passengers. After filling in the driver's box, the subject filled in the second box in the family car with the number five. This is because the subject assumes that there are six cars and has been chosen as driver 1, even though the number of people who can still be chosen is 8 people. This was also repeated for the selection of passengers in the second car. The subject assumes that the car's capacity is an option for passengers. So, in the second box, the subject entered three because the mini car quota was four and there was one less driver. This process is included in the Generate a number that represents the given aspects of the problem (CU1) category.

After that, the subject multiplied the numbers in each box on each car. This stage can be categorized as Identify the concept that fits the problem (FVa1) and the calculation process of the formula can be categorized as Calculating the results of operations on numbers that have been compiled (CU3).

Errors then appeared when the subject added up the multiplication of the numbers in the family car box and the mini car box. This error can be categorized as Identify the concept that fits the problem (FVa1). Based on this description, the flow of work on subject 2 can be seen in Figure 4.

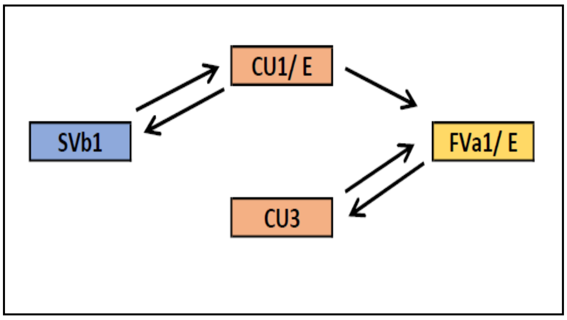

Figure 4. Flowchart of Subject 2 Process

There is a weak correlation between formula/expression and set of outcomes (Lockwood, 2013). These are two different ways. If students solve the problem through set of outcomes, there is little possibility of doing a formula/expression after. In this research, we colored up every category in the flowchart. Blue is color for set of outcomes, orange is color for counting process, and yellow is color for formula/expression. As a summary, the flowchart doesn't show a correlation between formula/expression and set of outcomes.

Filling slots is one of the most effective methods (Aini et al., 2018). This research finds several obstacles when students use this method. It is difficult for students to identify which number should be put in the boxes. Students should understand the problem clearly and then we can use these methods.

The obstacles that occur while students solve combinatorial problems. One of them is to find the appropriate concept and understand the problem (Sinaga et al., 2021). In this study, we find that students cannot decide between using the multiplication rule or the addition rule. Second, this research also finds out that students do not understand the problem clearly, so some mistakes occur while they are generating the number from the given.

CONCLUSION

This study has described the process of the two subjects solving combinatorial problems. The two subjects experienced similar errors. This error includes Generate a number that represents the given aspects of the problem and identify the concept that fits the problem. The research findings show that filling slots is a simple and easy way for students to understand, but problems arise when students cannot understand the meaning of the questions and input the correct numbers for the problem. The research findings also reinforce Lockwood's opinion (Lockwood, 2013) that there is a weak relationship between set of outcomes and formula/expression.

This study has limitations, one of which is the number of subjects and the variations in the subject's answers. Different errors may be found in different subjects. This research suggests enhancing students understanding of number representation when teaching the filling slot method. The teacher should illustrate some of the multiplication rule and addition rule examples to lead student distinguish between these two fundamental rules. Further research is needed to provide solutions to the constraints experienced by students in solving combinatorial problems.

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